Subject. (L.o5) Date.

Integration by Substitution

التكامل بالتعويض

[I] $\sqrt{ax+b}$ $\sqrt[3]{a+bx}$ - - - - -

Put $\sqrt{ax+b} = u \Rightarrow ax+b = u^2$

" $\sqrt[3]{ax+b} = u \Rightarrow ax+b = u^3$

EX: ① $\int \frac{1}{\sqrt{2x-1} + \sqrt[4]{2x-1}} dx = I$

put $2x-1 = u^4$

$2dx = 4u^3 du \Rightarrow dx = 2u^3 du$

$$I = \int \frac{1}{\sqrt{u^4} + \sqrt[4]{u^4}} \cdot 2u^3 du$$

$$= 2\int \frac{u^3}{u^2+u} du = 2\int \frac{u^2-1+1}{u+1} du = 2\int \frac{(u-1)\cancel{(u+1)}}{\cancel{(u+1)}} + \frac{1}{u+1}$$

$$= 2\int u-1 + \frac{1}{u+1} = 2\left[\frac{u^2}{2} - u + \ln|u+1|\right] + C$$

$\boxed{2}$ $\int \frac{1+\sqrt{x}}{1+\sqrt[3]{x}}\, dx$

put $x = u^6$

$dx = 6u^5\, du$

$I = \int \frac{1+u^3}{1+u^2} \cdot 6u^5\, du$

$= 6\int \frac{u^5+u^8}{1+u^2}\, du$

$\downarrow$

[illegible]

$= 6\int u^6 - u^4 + u^3 + u^2 - u - 1 + \frac{u+1}{u^2+1}\, du$

$= 6\left[\frac{u^7}{7} + \frac{-u^5}{5} + \frac{u^4}{4} + \frac{u^3}{3} - \frac{u^2}{2} - u + \frac{1}{2}\frac{2u}{u^2+1} + \frac{1}{u^2+1}\right.$

$\frac{1}{2}\ln|u^2+1|$ $\quad \tan^{-1} u$

$$
\begin{array}{r|l}
 & u^6 - u^4 + u^3 + u^2 - u - 1 \\
\hline
u^2+1 & u^8 + u^5 \\
 & \underline{-u^8 - u^6} \\
 & -u^6 + u^5 \\
 & \underline{+u^6 + u^4} \\
 & u^5 + u^4 \\
 & \underline{-u^5 - u^3} \\
 & u^4 - u^3 \\
 & \underline{-u^4 - u^2} \\
 & -u^3 - u^2 \\
 & \underline{+u^3 + u} \\
 & -u^2 + u \\
 & \underline{+u^2 + 1} \\
 & u + 1
\end{array}
$$

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$\boxed{3}$ $\int \sin(\sqrt{x})\, dx$

Put $\sqrt{x} = u \longrightarrow x = u^2$

$dx = 2u\, du$

$I = \int \sin(u) \cdot 2u\, du$

$= 2 \int u \sin(u)\, du$

u		$\sin u$
	$+$	
1		$-\cos u$
	$-$	
0		$-\sin u$
	$+$	

$I = -u \cos u + \sin u + C$

$u = \sqrt{x}$

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$\boxed{4}$ $\int \frac{\cos\sqrt{x+1}}{\sqrt{x+1}}\,dx$

Put $\sqrt{x+1} = u \Rightarrow x+1 = u^2$

$dx = 2u\,du$

$I = \int \frac{\cos u}{\cancel{u}}\, 2\cancel{u}\,du$

$= 2\int \cos u\,du$

$= 2\sin u + C$

$= 2\sin\sqrt{x+1} + C$

$\boxed{5}$ $\int \frac{e^x}{1+e^{2x}} dx$

$u = e^x$

$du = e^x dx$

$\int \frac{du}{1+u^2} = \tan^{-1} u + c$

$= \tan^{-1}(e^x) + C$

$\boxed{6}$ $\int \frac{\sin(2\ln x)}{x} dx$

$u = 2\ln x$

$du = \frac{2}{x} dx$

$I = \frac{1}{2} \int \frac{2\sin(2\ln x)}{x} dx$

$= \frac{1}{2} \int \sin u \, du$

$= -\frac{1}{2} \cos u + C$

$= -\frac{1}{2} \cos 2\ln x + C$

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تكاملات على الصوره *

[1] $\int \sqrt{a^2-x^2}\, dx \Longrightarrow x = a\sin(\theta)$

[2] $\int \sqrt{x^2-a^2}\, dx \Longrightarrow x = a\sec(\theta)$

[3] $\int \sqrt{x^2+a^2}\, dx \Longrightarrow x = a\tan(\theta)$

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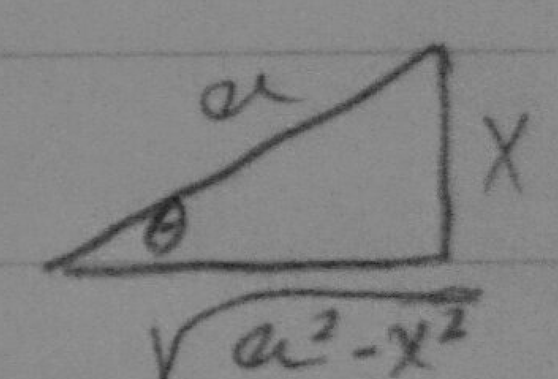
[1] $\int \sqrt{a^2-x^2}\, dx$

put $x = a\sin\theta$

$dx = a\cos\theta\, d\theta$



$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\frac{x}{a}\cdot\frac{\sqrt{a^2-x^2}}{a}$

$= \frac{2x\sqrt{a^2-x^2}}{a^2}$

$$I = \int \sqrt{a^2 - a^2\sin^2\theta}\cdot a\cos\theta\, d\theta$$

$$= a^2\int\sqrt{1-\sin^2\theta}\cdot\cos\theta\, d\theta$$

$$= a^2\int\sqrt{\cos^2\theta}\cdot\cos\theta\, d\theta = a^2\int\cos^2\theta\, d\theta$$

$$= a^2\int\frac{1}{2}(1+\cos 2\theta)\, d\theta = \frac{a^2}{2}\int(1+\cos 2\theta)\, d\theta$$

$$= \frac{a^2}{2}\left[\theta + \frac{\sin 2\theta}{2}\right] + C \quad , \theta = \sin^{-1}\frac{x}{a}$$

$$= \frac{a^2}{2}\left[\sin^{-1}\frac{x}{a} + \frac{2x\sqrt{a^2-x^2}}{2a^2}\right] + C$$